Quantum algorithms for the property testing of Boolean functions

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A Boolean function maps n-bit strings to either 0 or 1. Suppose one is given an unknown Boolean function in the form of a black box, one gives the box an input x, and its output is f(x), and it is guaranteed that the function has a particular property or is a distance epsilon from having that property (the distance between two Boolean functions is the fraction of inputs on which they have differing outputs). One wants to determine which of the two alternatives obtains with as few function calls as possible. Typical properties are linearity or dictatorship (dependence on only one of the n bits). We give quantum algorithms for testing two properties, linearity and permutation invariance of the arguments, that are better than known classical algorithms. Classical algorithms can decide whether the function has this property or is far from having it in (\epsilon)^{-1} function calls, while the quantum algorithms only require (\epsilon)^{-2/3} function calls. Our algorithms make use of known quantum algorithms, the Bernstein-Vazirani algorithm and the Grover algorithm.

References

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