Strategies for optimum state discrimination and their mutual relations

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State discrimination is the task to determine the actual state of a quantum system, prepared with given prior probabilities in one of N known states. Since quantum states cannot be distinguished perfectly unless they are mutually orthogonal, discrimination strategies have been developed, which are optimized with respect to various criteria. The optimum measurements performing these strategies are often generalized measurements, and in many cases they require a certain probability of getting an inconclusive outcome.

The specific optimum measurement that discriminates the states with the maximum overall probability of correct results, while the probability of inconclusive results is fixed at a given value, plays a central role. When the fixed value increases, starting from zero, the optimum measurement interpolates between the strategies of minimum-error discrimination and a measurement that under certain conditions corresponds to the strategy of optimized maximum-confidence discrimination, or of optimum unambiguous discrimination, respectively. Provided that analytical solutions of this specific optimization problem are possible, they can be derived from operator conditions [1] determining the optimum measurement. In the talk complete analytical solutions are presented for cases considered previously [2] in the context of optimized maximum-confidence discrimination, in particular for the discrimination of states that belong to the class of two mixed qubit states, including two pure states occurring with arbitrary prior probabilities, or to the class of N symmetric qudit states, both pure and mixed. As outlined in the talk, if the optimum measurement is known for an arbitrary value of the fixed probability of inconclusive results, then this complete solution also determines the optimum measurement in another strategy, where the overall error probability is fixed, and vice versa.

[1] J. Fiurášek and M. Ježek, Phys. Rev. A **67**, 012321 (2003).
[2] U. Herzog, Phys. Rev. A **79**, 032323 (2009), ~, Phys. Rev. A **85**, 032312 (2012).